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ABSTRACT

The analysis of interaction effects in multiple regression has received considerable attention in recent years, but problems with the valid identification of moderating variables have been noted by researchers. G. McClelland and C. Judd (1993), in their discussion of the statistical difficulties of detecting interactions and moderating effects, warned against the use of a four-corners subsample approach to moderated multiple regression, but they did not present empirical evidence that such an approach provides less power than the use of the full random sample. This study was conducted to produce evidence of the extent of power loss that is associated with the subsample strategy. The effectiveness of the four-corners subsample procedure was investigated through a Monte Carlo study that used regression models to generate data from populations with linear, nonlinear, and nonadditive relationships. In all, 2,304 conditions were examined, for 3 models, 4 levels of population "R" squared, 4 levels of regressor correlation, 4 levels of regressor reliability, 3 levels of sample size, and 4 levels of effect size for the nonlinear or nonadditive component. Results suggest that the use of the four-corners strategy rather than full sample analysis shows better specificity at the expense of reduced statistical power, or sensitivity, relative to full sample analysis. Despite the improved specificity of the four-corners approach, model misidentification rates were high in many of the conditions examined. The utility of either the four-corners approach or the full sample approach for testing theory is limited. (Contains 3 tables and 26 references.) (SLD)

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The McClelland and Judd Approach: Using "Four-corners" Data to Detect Nonlinearity and Nonadditivity

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The McCleland and Judd Approach:
Using "Four-corners" Data to Detect Nonlinearity and Nonadditivity

As Manly (1992) noted,.."in a multiple regression analysis, a single variable y is related to two or more variables to see how Y is related to the X's" (1992). Problems can arise, however, while interpreting results of data collected from observations that are not like those from the population of interest (i.e., an atypical sample); or by failing to correctly specify the functional form of the relationship between the predictors and the criterion variable. Two examples of the latter are nonlinearity and nonadditivity. Nonlinearity in the model occurs when the regression of y on at least one X variable depends upon the value of that variable (either accelerating or decelerating. Cortina (1993) relates "... the possibility of nonlinear relationships continues to go relatively unexplored. For this reason, interpretation of significant interaction terms in multiple regression may be difficult ..." Budescu (1980) reported, "as the degree of collinearity increases, the results of the analysis become more and more a function of the internal relations between the predictors...''. Saunders (1955) was first to devise a method to test interactions (moderator effects) and called his invention "moderated multiple regression." Customarily, it is necessary to test for an interaction (moderator effect) when the effect of one variable, X, on a second variable, y, seems to depend on the level of a third variable, z. The problem of nonadditivity in the model refers to a product term consisting of two predictors multiplied together; creating a joint effect of these independent variables on the dependent variable. Nonlinearity and nonadditivity are referred to as specification errors when there is a lack of proper congruence between the sample regression model and the population. There exist both true (correctly specified) and misspecified models. "The rub, however, is that the true model is seldom, if ever , known" (Pedhazur,1982).

Detecting Nonlinearity and Nonadditivity

The analysis of interaction effects in multiple regression has received considerable attention in recent years (e.g., Aiken & West, 1991; Jaccard & Wan, 1995; McClelland & Judd, 1993), although the methods for such analyses have been known for at least 40 years (Saunders, 1955). An interaction effect indicates that the relation between a criterion variable (Y) and a predictor variable (X) varies as a function of some third variable (Z). This third variable is commonly referred to as a moderator (Saunders, 1955). Moderator variables are common in behavioral research (Baron & Kenny, 1986). For example, Perlin, Menagham, Lieberman, and Mullen (1981) hypothesized moderating effects for both coping responses and social support, on the relationship between stressful events and health. Similarly, Findley and Cooper (1983) hypothesized that the relationship between locus of control and academic achievement is moderated by demographic factors such as gender, race, and socio-economic status.

A statistical test for interaction (or moderator) effects is usually accomplished with hierarchical multiple regression, in which differences in sample R^2 values between an additive model and a non- additive model are tested (equivalently, for a single moderator component, the test of the regression weight for the product term may be used). Although alternative testing procedures have been recommended in the literature, such procedures subsequently have been shown to be incorrect (e.g., Cronbach, 1987; Dunlap & Kemery, 1987). As McClelland and Judd (1993) asserted, there has been "no credible published refutation of the appropriateness of [hierarchical multiple regression] as a test of moderator effects" (p. 377).

Problems with the valid identification of moderating variables have been noted by both research methodologists and applied researchers. Some of the more frequently encountered

difficulties in statistically detecting such effects in non-experimental research have been attributed to measurement error (Dunlap & Kemery, 1988; Jaccard & Wan, 1995), multicollinearity (Morris, Sherman & Mansfield, 1986), low residual variance of the product term in the regression equation (McClelland & Judd, 1993), residual variance heterogeneity (Alexander & DeShon, 1994), and even a natural consequence of multivariate normality (Fisicaro & Tisak, 1994).

McClelland and Judd (1993) noted the relative ease with which interaction effects are apparently detected in experimental research, in contrast with the difficulties of their detection in field studies. These authors attributed the power deficits seen in field research to a lack of residual variance in the product term used in moderated multiple regression, an effect attributable to the use of nonoptimal distributions of regressor variables in field research. That is, experimental research is characterized by observations occurring at extreme values of the regressor variables, while field research is characterized by observations occurring at more moderate values.

McClelland and Judd (1993) clearly warned against naive applications in field research of their ideas. For example, artificially dichotomizing regressor variables does not make the observations on those variables ***truely*** extreme. Further, Maxwell and Delaney (1993) demonstrated that such dichotomization can easily distort the relationships between variables. A second "unwise strategy" noted by McClelland and Judd is the collection of a random sample of data in field research from which an approximately optimal subsample is obtained. This subsample of data, from the four corners of the bivariate distribution, is then analyzed using moderated multiple regression. Although intuitively appealing to some extent, the use of such a

subsample is likely to lead to even less statistical power (because of the smaller sample size) than that obtained from the random sample itself (despite the nonoptimal distribution in the random sample).

Although McClelland and Judd (1993) warned against the use of a 4-corners subsample approach to moderated multiple regression, they did not present empirical evidence that such an approach provides less power than the use of the full random sample. The present study was designed to produce evidence of the extent of power loss that is associated with the subsample strategy.

A Variety of Potential Regression Models

Consider a multiple regression equation, with two regressors X and Z. If the linear regression model, $y = \alpha + \beta_1 X + \beta_2 Z + \epsilon$, is fit to a set of data, and inspection of (for example) partial regression plots indicates departure from linearity, the researcher is not certain which of the following models may accurately describe the relationship between the regressors and the dependent variable:

a) $y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$ (moderation)

b) $y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 X^2 + \epsilon$ (nonlinearity in X)

c) $y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 Z^2 + \epsilon$ (nonlinearity in Z)

d) $y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 X^2 + \beta_4 Z^2 + \epsilon$ (nonlinearity in X and Z)

e) $y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 X^2 + \beta_4 Z^2 + \beta_5 XZ + \epsilon$ (nonlinearity and nonadditivity)

If a researcher lacks a theoretical reason for expecting a particular functional form of the relationship, inspection of each model may be made to determine whether the nonlinear model or

the moderation model better describes the relationship between the regressors and the dependent variables. The underlying relationship may be best represented by a moderated equation (model a), suggesting that the relationship between the outcome variable and each of the regressors depends on the value of the other regressor. In contrast, the underlying relations may be best represented by a nonlinear relationship between one of the regressors and the criterion variable (models b and c), by nonlinear relationships between both regressors and the criterion (model d), or by a combination of nonlinearity and moderation (model e).

Selection of the wrong model based upon sample data may be considered a Type I error, a Type II error, or a lack of specificity of the test used. For example, if the population from which the sample was drawn is accurately characterized by a linear, additive model, the selection of any nonlinear or nonadditive model (a through e) represents a Type I error. Conversely, a Type II error may result if the population is best characterized by a nonlinear or nonadditive model, but none of the nonlinear or nonadditive models provide a sufficient increase in R^2 relative to the additive model. Finally, if the population is best characterized by a nonlinear model, but the researcher selects a moderated model based upon the sample data, a lack of specificity is evident. A test with good specificity will lead to rejecting the null hypothesis associated with the actual population model, but not rejecting null hypotheses associated with other models.

A number of factors are related to the lack of specificity in moderated multiple regression, but probably the greatest contributor to such errors is the presence of measurement error in the instruments used to represent the phenomenon being investigated. The importance of measurement error in selecting the best-fitting model from competing models has been discussed previously in great detail (cf., Busemeyer & Jones, 1983; MacCallum & Mar, 1995), particularly

for those models that incorporate multiplicative composite terms. Essentially, the reliability of the product terms is a joint function of the reliabilities of the components (X and Z) and the correlation between the components.

If X and Z are not correlated, then the reliability of the product term (XZ) is equal to the product of the separate reliabilties of X and Z. Thus, if the separate reliabilities of the component terms are relatively high, then the reliability of the product term will be high. Conversely, low component reliabilities will result in low composite reliability scores, and an increased probability of committing a Type II error by not detecting an effect when one is present. As the reliability of each component increases, the reliability of the composite increases and the Type II error probability decreases.

This phenomena is also true of nonlinear effects. As has been pointed out by Shepperd (1991), the quadratic composite term in a regression model would also suffer from unreliability if the component terms were unreliable, the reliability of the composite term X^2 being equal to the square of the reliability of X. The effects of such unreliability on the quadratic term are the same as the effects on the cross-product term noted above -- a reduction in statistical power and a concomitant increase in the probability of a Type II error.

The reliability problem is compounded when the regressor variables are correlated with each other. As has been noted many times, as the correlation between X and Z increases, the quadratic term (X^2) and the interaction term (XZ) will share substantial variance and will become more difficult to differentiate.

Method

The effectiveness of the 4-corners subsample procedure was investigated through a Monte Carlo study which used regression models to generate data from populations evidencing (a) linear, (b) non-linear, and (c) non-additive relationships. In addition to the functional form of relationship characteristic of the population, five factors were manipulated in the study: (a) the correlation between the regressor variables, (b) the overall population R^2 of the additive component of the regression model, (c) the effect size of the non-linear or interaction terms (X^2, Z^2, and XZ), (d) the reliabilities of the regressors, and (e) sample size. Only models with two regressor variables were included in the study.

The magnitude of the correlation between the two regressors was controlled at levels ranging from .00 to .90. Four levels of R^2 of the additive component of the population models were examined: .02, .13, .26, and .50. The first three levels represent small, medium, and large effect sizes for the population R^2, corresponding to f^2 values of .02, .15, and .35 (Cohen, 1988). The population R^2 value of .50 was included based upon the review of correlational studies conducted by Jaccard and Wan (1995). In this review, the 75th percentile of the distribution of sample R2s found in the psychological literature was .50. The magnitudes of the interaction component or the non-linearity component were controlled at four levels, representing small, medium, and large effect sizes (Cohen, 1988), as well as a null condition.

Measurement error was simulated in the data (following the procedure used by Maxwell, Delaney & Dill, 1984; and by Jaccard & Wan, 1995) by generating four normally distributed random variables for each observation (two to represent "true scores" on the regressors, and two to represent errors of measurement). Fallible, observed scores on the regressors were calculated

(under "classical" measurement theory) as the sum of the true and error variables. The reliabilities of the regressors were controlled by adjusting the error variances relative to the true score variances. Reliabilities were examined ranging from .40 to 1.00.

Sample sizes of 60, 175 and 400 were used. The larger two of these values represent the median and 75th percentile of sample sizes found in Jaccard and Wan's (1995) review of correlational studies in psychology. The small sample size (n = 60) was included to extend the results to small sample analyses. Five thousand samples of each size were generated for each condition in the Monte Carlo study. The use of five thousand replications provide maximum 95% confidence intervals of ± .014 around the observed proportion of null hypotheses rejected.

For each sample, the entire sample was analyzed using moderated multiple regression, then a 4-corners subsample was extracted. The subsample was selected by retaining only the most extreme 10% of the observations from each corner of the sample bivariate distribution. The 4-corners subsample was then analyzed using moderated multiple regression. The moderated multiple regression strategy involved fitting four models to each sample (and each 4-corners subsample): a linear additive model, a model nonlinear in X_1, a model nonlinear in X_2, and a nonadditive model. Tests for the presence of nonlinearity and nonadditivity were conducted by testing the statistical significance of the nonlinear or nonadditive term.

The Monte Carlo study was conducted using SAS, Versions 6.06 and 6.08. The components of the program were verified by comparing the results with the standard SAS output for benchmark data sets.

Results and Discussion

In total, 2304 conditions were examined in the Monte Carlo study (i.e., three models, four levels of population R^2, four levels of regressor correlation, four levels of regressor

reliability, three levels of sample size, and four levels of effect size for the nonlinear or nonadditive component). To conserve space, and because the results were substantively consistent across levels of these design factors, only summary results will be presented here. Complete results, however, are available from the authors.

The results will be presented in terms of the proportion of samples in which the correct population model was identified (i.e., a linear-additive model, a nonadditive model, or a model nonlinear in X_1), and the proportion of samples in which an incorrect model was identified. Such proportions are related to, respectively, the sensitivity and the specificity of these analysis strategies. Because the tests for nonlinearity and nonadditivity were conducted independently of each other, an individual sample may lead to a rejection of more than one of the null hypotheses. Thus, a single sample may suggest either nonlinearity or nonadditivity.

The results for the linear-additive models are presented in Table 1. This table presents, each nominal alpha level, the proportion of samples that were identified as evidencing nonadditivity or nonlinearity in either X_1 or X_2. Any of these model identifications represent Type I errors in the rejection of the null hypothesis of no change in the model R^2 relative to a linear-additive model. As is evident in this table, the Type I error rate was well controlled whether the complete sample was analyzed or whether the 4-corners of the data were used. In each condition, and for each nominal alpha level, the proportion of samples that led to a rejection of the null hypothesis was very close to the nominal level of alpha. In addition to verifying the Type I error control of these methods of analysis, these results provide a check on the integrity of the computer code written for the Monte Carlo study.

Insert Table 1 about here

The results for the nonadditive population models are presented in Table 2. The proportions presented in this table represent either statistical power (i.e., the proportion of samples in which the nonadditive model was identified based upon the sample data), or misidentification rates (i.e., the proportion of samples in which each of the nonlinearity null hypotheses was rejected). For example, the first row of Table 2 reports the overall results with a population R^2 of .02 when data were generated from a nonadditive population model. For these samples, with a nominal alpha level of .10, the moderated regression model was identified in 72.9% of the samples when all of the sample data were included in the regression (i.e., a power estimate of .729). However, in 40.3% of these samples, a regression model that was nonlinear in X_1 also fit the data statistically significantly better than the linear-additive model, and in 40.6% of the samples a model that was nonlinear in X_2 also fit better than the linear-additive model. Thus, researchers testing hypotheses about these models would misidentify the population model at rates of greater than .40. In contrast, when only the 4-corners of the samples are used for the regression analyses, the correct model was identified in 70.1% of the samples, providing slightly less power than was obtained with the use of the full samples. However, the estimated rate of misidentifying the model as nonlinear were also lower than those obtained with the full samples (giving estimates of .360 for nonlinear X^1 and .361 for nonlinear X^2. Both the statistical power and Type I error rate estimates remained relatively stable across levels of R^2 in the population. In each case, the use of the full samples provided greater statistical power, but such power

advantages were accompanied by greater probabilities of misidentifying the population as nonlinear.

Insert Table 2 about here

A similar result was obtained for the level of correlation between the regressors. Across all levels, the use of the full model provided greater statistical power but higher misidentification rates. In contrast the effects of population R^2, the level of correlation between the regressors affected both the power and the Type I error rates of these tests. Specifically, as the correlation between the regressors increased, the statistical power increased for both full sample and four-corners strategies. However, a concomitant decrease in the specificity was also evident for both strategies. For example, at a nominal alpha level of .05, when the regressors were uncorrelated, the power of the full sample analysis was .614 while that of the 4-corners analysis was .591. Both analysis strategies also evidenced low levels of model misidentification, with the nonlinear X_1 model identified only 8.5% of the time with the full samples and only 7.7% of the time with the 4-corners. In contrast, when the level of correlation between the regressors was .80, the power to detect the moderating model increased to .733 for the full samples and to .680 for the 4-corners. However, the misidentification rate increased to 64.5% for the full samples and to 56.9% for the four corners.

The effect of regressor reliability was similar to that of regressor intercorrelation although the effect was smaller in magnitude. That is, increasing regressor reliability led to increasing the power in the test for the moderating model, but also led to increasing

misidentification rates. For example, at a nomina alpha level of .05, when the regressor reliability was .40, the power of the full sample analysis was .452 while that the 4-corners analysis was .413. Both analysis strategies also evidenced relatively low levels of model misidentification, with the nonlinear X_1 model identified only 20.0% of the time with the full samples and only 14.8% of the time with the 4-corners. In contrast, when the reliability of the regressors was 1.00, the power to detect the moderating model increased to .814 for the full samples and to .789 for the 4-corners. However, the misidentification rate increased to 44.6% for the full samples and to 41.3% for the four-corners.

Finally, as should be expected, increasing sample sizes or increasing the population effect sizes led to increases in power of the test for the moderated regression model, but concommittant increases in the rates of misidentifying the model. Such effects were evident for both the full samples analyses and the 4-corners analyses. In all conditions, however, the power of the full sample analyses was greater than that of the four-corners.

The results for the nonlinear X_1 population models are presented in Table 3. As with the results presented in Table 2, the proportions presented in this table represent either statistical power (i.e., the proportion of times in which the nonlinear X_1 model was identified based upon the sample data), or misidentification rates (i.e., the proporation of samples in which the null hyupothesis of nonadditivity or the null hypothesis of nonlinearity in X^2 were rejected).

Insert Table 3 about here

The results for the nonlinear population models were nearly identical to those for the nonadditive models. That is, the use of the full samples was assocaited with greater statistical power but also with higher rates of misidentification. For both the full sample and the 4-corners strategies, the power increased with increasing correlation between regressors, with increasing reliability of regressors, with increasing sample size and with increasing effect size. However, while these factors lead to greater statistical power, they also led to higher rates of misidentification.

The results suggest that the use of the 4-corners strategy rather than the full sample analysis has both benefits and costs. Specifically, the 4-corners strategy evidenced better specificity (i.e., lower misidentification rates), but at the expense of reduced statistical power , or sensitivity, relative to the full sample analysis. Despite the improved specificity of the 4-corners approach, the model misidentifacation rates were distressingly high in many of the conditions examined. With increasing regressor intercorrelations, increasing reliablility of regressors, increasing sample size and increasing effect size, the probabilities of rejecting null hypotheses associated with the incorrect functional form of the model increased along with the statistical power of the test for the correct functional form. Thus, the utility of either the 4-corners approach or the full sample approach for testing theory is limited.

For example, in applied research, if a particular theory suggests that a nonadditive model should be present in a population, researchers may collect a sample of data from that population and test the null huypothesis of nonadditivity in the sample. If the theory is correct, the use of the full sample will lead to a greater chance of identifying the nonadditivity (i.e., researchers will have more statistical power by using the full sample rather than the four-corners). However, if

the theory is wrong and the population actually evidences nonlinearity rather than nonadditivity, then the use of the full sample will lead to a greater chance of misidentifying the model as nonadditive. That is the full sample analysis is more likely to provide support for a theory which is wrong, while the subsample approach is less likely to support such a theory. However, in many conditions neither approach should provide prudent researchers with much confidence that the correct model has been identified.

REFERENCES

Aiken, L. & West, S. (1991). Multiple regression. Newbury Park, CA: Sage.

Alexander, R. A. & DeShon, R. P. (1994). Effect of error variance heterogeneity on the power of tests for regression slope differences. Psychological Bulletin, 115, 308-314.

Bradley, R.A., & Srivastava, S.S. (1979). Correlation in polynomial regression. The American Statistician, 33, 11-14.

Budescu, D.V. (1980). A note on polynomial regression. Multivariate Behavioral Research, 15, 497-508.

Busemeyer, J. R. & Jones, L. E. (1983). Analysis of multiplicative combination rules when the causal variables are measured with error. Psychological Bulletin, 93, 549-562.

Cohen, J. (1988). Statistical power analysis for the behavioral sciences (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.

Cohen, J. (1978). Partialled products are interactions; Partialled powers are curve components. Psychological Bulletin, 85(4), 858-866.

Cortina, J. (1993). Interaction, nonlinearity, and multicollinearity: Implications for multiple regression. Journal of Management, 14(4), 915-922.

Cronbach, L. J. (1987). Statistical tests for moderator variables: Flaws in analyses recently proposed. Psychological Bulletin, 102, 414-417.

Dunlap, W. P. & Kemery, E. R. (1987). Failure to detect moderating effects: Is multicollinearity the problem? Psychological Bulletin, 102, 418-420.

Dunlap, W. P. & Kemery, E. R. (1988). Effects of predictor intercorrelations and reliabilities on moderated multiple regression. Organizational Behavior and Human Decision Processes, 41, 248-258.

Findley, M. J. & Cooper, H. M. (1983). Locus of control and academic achievement: A literature review. Journal of Personality and Social Psychology, 44, 419-427.

Fisicaro, S. A. & Tisak, J. (1994). A theoretical note on the stochastics of moderated multiple regression. Educational and Psychological Measurement, 54, 32-41.

Jaccard, J. & Wan, C. K. (1995). Measurement error in the analysis of interaction effects between continuous predictors using multiple regression: Multiple indicator and structural equation approaches. Psychological Bulletin, 117, 348-357.

MacCallum, R. C. & Mar, C. M. (1995). Distinguishing between moderator and quadratic effects in multiple regression. Psychological Bulletin, 118, 405-421.

Manly, B.F., (1994). Multivariate statistical methods: A primer. London, United Kingdom: Chapman & Hall.

Maxwell, S. E., Delaney, H. D., & Dill, C. A. (1984). Another look at ANCOVA versus blocking. Psychological Bulletin, 95, 136-147.

McClelland, G.H., & Judd, C.M. (1993). Statistical difficulties of detecting interactions and moderator effects. Psychological Bulletin, 114(2), 376-390.

Morris, J. H., Sherman, J. D., & Mansfield, E. R. (1986). Failures to detect moderating effects with ordinary least squares -- moderated multiple regression: Some reasons and a remedy. Psychological Bulletin, 99, 282-288.

Pedhazur, E. (1982). Multiple regression in behavioral research, (2nd ed.). New York, NY: CBS College Publishing.

Perlin, L. I., Menagham, E. G., Lieberman, M. A. & Mullan, J. T. (1981). The stress process. Journal of Health and Social Behavior, 22, 337-356.

Pooch, U.W., & Wall, J.A. (1993). Discrete event simulation: A Practical approach. Boca Raton, FL: CRC Press.

SAS Institute, Inc., (1996). Multivariate statistical methods: Practical applications. Carey, N.C.: SAS Institute, Inc.

Saunders, D.R. (1955). The "Moderator Variable" as a useful tool in prediction. Proceedings of the 1954 Invitational conference on Testing Problems, 54-58. Princeton, NJ: Educational Testing Service.

Shepperd, J. A. (1991). Cautions in assessing spurious "moderator effects." Psychological Bulletin, 110, 315-317.

Thompson, B. (January, 1989). Heuristics for understanding the concepts of interaction, polynomial trend, and the general linear model. Paper presented at the annual meeting of of the Southwest Educational Research Association, Houston, Texas.

Table 1
Summary Results for Linear Population Models.

Population R-Square	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
.02	0.100	0.101	0.100	0.100	0.101	0.101	0.050	0.051	0.050	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
.13	0.099	0.099	0.099	0.100	0.099	0.100	0.050	0.050	0.049	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
.26	0.099	0.099	0.101	0.100	0.100	0.101	0.049	0.049	0.050	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
.50	0.099	0.099	0.101	0.099	0.099	0.101	0.049	0.049	0.050	0.050	0.049	0.050	0.010	0.010	0.010	0.010	0.010	0.010

Correlation Between Regressors	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
.00	0.100	0.100	0.099	0.100	0.099	0.100	0.049	0.049	0.050	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
.20	0.100	0.100	0.100	0.100	0.100	0.100	0.050	0.050	0.051	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
.40	0.099	0.099	0.100	0.100	0.100	0.101	0.049	0.049	0.049	0.050	0.050	0.051	0.010	0.010	0.010	0.010	0.010	0.010
.80	0.100	0.099	0.101	0.100	0.099	0.101	0.050	0.050	0.050	0.050	0.049	0.050	0.010	0.010	0.010	0.010	0.010	0.010

Regressor Reliability	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
0.40	0.100	0.099	0.100	0.100	0.099	0.101	0.050	0.050	0.050	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
0.60	0.100	0.099	0.101	0.100	0.100	0.100	0.050	0.049	0.050	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
0.80	0.099	0.100	0.100	0.099	0.100	0.100	0.049	0.049	0.050	0.049	0.050	0.049	0.010	0.010	0.010	0.010	0.009	0.010
1.00	0.099	0.099	0.100	0.100	0.100	0.101	0.049	0.050	0.049	0.051	0.050	0.051	0.010	0.010	0.010	0.010	0.010	0.010

Sample Size	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
60	0.099	0.099	0.100	0.100	0.099	0.101	0.050	0.049	0.050	0.050	0.050	0.050	0.010	0.010	0.010	0.010	0.010	0.010
175	0.100	0.100	0.100	0.100	0.100	0.101	0.049	0.050	0.050	0.050	0.050	0.051	0.010	0.010	0.010	0.010	0.010	0.010
400	0.099	0.099	0.100	0.100	0.099	0.099	0.050	0.049	0.050	0.050	0.049	0.049	0.010	0.010	0.010	0.010	0.010	0.010

Note. For models identified, Mod = nonadditive model, X_1^2 = nonlinear X_1 model, X_2^2 = nonlinear X_2 model

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Table 2
Summary Results for Nonadditive Population Models.

Population R-Square	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
.02	0.729	0.406	0.406	0.701	0.360	0.361	0.671	0.337	0.337	0.637	0.289	0.290	0.561	0.238	0.238	0.516	0.191	0.191
.13	0.723	0.403	0.403	0.695	0.358	0.358	0.664	0.334	0.334	0.630	0.287	0.287	0.553	0.235	0.235	0.509	0.189	0.189
.26	0.715	0.398	0.398	0.687	0.354	0.354	0.656	0.329	0.329	0.622	0.284	0.284	0.545	0.231	0.231	0.500	0.186	0.187
.50	0.692	0.386	0.384	0.664	0.345	0.344	0.631	0.317	0.316	0.597	0.274	0.274	0.517	0.219	0.219	0.475	0.178	0.178

Correlation Between Regressors	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
.00	0.678	0.147	0.146	0.659	0.137	0.137	0.614	0.085	0.084	0.591	0.077	0.077	0.496	0.025	0.025	0.467	0.021	0.021
.20	0.687	0.274	0.273	0.666	0.236	0.236	0.624	0.198	0.197	0.598	0.163	0.162	0.508	0.099	0.099	0.474	0.072	0.072
.40	0.711	0.467	0.467	0.683	0.405	0.405	0.651	0.390	0.389	0.617	0.326	0.326	0.539	0.268	0.267	0.495	0.206	0.206
.80	0.783	0.705	0.705	0.739	0.638	0.639	0.733	0.645	0.645	0.680	0.569	0.570	0.633	0.532	0.533	0.565	0.446	0.446

Regressor Reliability	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
0.40	0.531	0.271	0.271	0.497	0.218	0.217	0.452	0.200	0.199	0.413	0.148	0.148	0.318	0.110	0.110	0.274	0.068	0.067
0.60	0.687	0.366	0.366	0.654	0.314	0.314	0.621	0.294	0.294	0.583	0.241	0.241	0.502	0.194	0.195	0.453	0.144	0.144
0.80	0.787	0.445	0.445	0.761	0.404	0.404	0.735	0.376	0.376	0.702	0.332	0.332	0.629	0.273	0.274	0.584	0.226	0.227
1.00	0.854	0.511	0.510	0.835	0.482	0.482	0.814	0.446	0.445	0.789	0.413	0.413	0.727	0.346	0.345	0.690	0.307	0.307

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Note. For models identified, Mod = nonadditive model, X_1^2 = nonlinear X_1 model, X_2^2 = nonlinear X_2 model

Table 2 (Continued)
Summary Results for Nonadditive Population Models.

Sample Size	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
60	0.530	0.279	0.278	0.487	0.241	0.241	0.450	0.207	0.206	0.398	0.169	0.169	0.312	0.114	0.114	0.248	0.080	0.080
175	0.745	0.405	0.405	0.719	0.359	0.359	0.687	0.335	0.334	0.656	0.287	0.287	0.575	0.233	0.232	0.534	0.187	0.187
400	0.869	0.510	0.510	0.854	0.462	0.463	0.830	0.446	0.446	0.811	0.395	0.395	0.745	0.346	0.346	0.718	0.292	0.293

Effect Size	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
0.02	0.422	0.219	0.219	0.392	0.199	0.199	0.329	0.149	0.149	0.298	0.130	0.131	0.183	0.066	0.066	0.156	0.053	0.053
0.15	0.815	0.442	0.439	0.783	0.393	0.391	0.761	0.370	0.368	0.722	0.319	0.318	0.646	0.262	0.262	0.593	0.212	0.212
0.35	0.907	0.534	0.535	0.885	0.471	0.472	0.876	0.469	0.470	0.845	0.402	0.402	0.803	0.365	0.365	0.752	0.294	0.294

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Note. For models identified, Mod = nonadditive model, X_1^2 = nonlinear X_1 model, X_2^2 = nonlinear X_2 model

Table 3
Summary Results for Nonlinear Population Models.

Population R-Square	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
.02	0.504	0.779	0.262	0.404	0.678	0.248	0.443	0.728	0.201	0.333	0.613	0.182	0.351	0.628	0.130	0.229	0.492	0.103
.13	0.465	0.776	0.261	0.404	0.677	0.249	0.397	0.726	0.201	0.333	0.612	0.181	0.295	0.627	0.128	0.229	0.493	0.102
.26	0.460	0.770	0.259	0.400	0.670	0.246	0.392	0.718	0.198	0.329	0.605	0.179	0.290	0.618	0.126	0.225	0.485	0.101
.50	0.445	0.748	0.253	0.388	0.650	0.240	0.377	0.694	0.193	0.316	0.583	0.174	0.276	0.592	0.121	0.215	0.463	0.097

Correlation Between Regressors	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
.00	0.195	0.765	0.100	0.141	0.652	0.182	0.131	0.712	0.050	0.081	0.586	0.111	0.064	0.611	0.010	0.022	0.466	0.034
.20	0.352	0.768	0.112	0.266	0.658	0.142	0.276	0.717	0.059	0.188	0.592	0.079	0.170	0.616	0.014	0.088	0.470	0.020
.40	0.556	0.768	0.205	0.472	0.666	0.122	0.484	0.717	0.135	0.390	0.600	0.066	0.362	0.616	0.054	0.258	0.480	0.017
.80	0.771	0.772	0.619	0.716	0.698	0.537	0.718	0.721	0.549	0.653	0.635	0.460	0.616	0.621	0.427	0.531	0.518	0.331

Regressor Reliability	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
0.40	0.333	0.580	0.189	0.291	0.453	0.160	0.261	0.506	0.128	0.218	0.369	0.098	0.162	0.376	0.061	0.123	0.233	0.035
0.60	0.432	0.743	0.241	0.376	0.630	0.219	0.362	0.684	0.180	0.302	0.557	0.153	0.257	0.570	0.108	0.198	0.427	0.077
0.80	0.544	0.843	0.283	0.438	0.751	0.276	0.484	0.800	0.223	0.368	0.690	0.208	0.390	0.706	0.150	0.262	0.572	0.125
1.00	0.565	0.907	0.322	0.491	0.842	0.328	0.503	0.878	0.262	0.423	0.798	0.257	0.402	0.813	0.186	0.316	0.702	0.166

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Note. For models identified, Mod = nonadditive model, X_1^2 = nonlinear X_1 model, X_2^2 = nonlinear X_2 model

Table 3 (Continued)
Summary Results for Nonlinear Population Models.

Sample Size	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
60	0.330	0.593	0.190	0.269	0.464	0.169	0.256	0.519	0.129	0.194	0.379	0.106	0.153	0.390	0.061	0.096	0.240	0.039
175	0.479	0.801	0.262	0.407	0.697	0.245	0.412	0.749	0.201	0.333	0.631	0.178	0.308	0.645	0.128	0.227	0.506	0.100
400	0.596	0.911	0.324	0.521	0.845	0.323	0.539	0.882	0.265	0.456	0.800	0.253	0.447	0.814	0.189	0.352	0.704	0.163

Effect Size	Alpha = .10						Alpha = .05						Alpha = .01					
	Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis			Full Sample Analysis			Subsample Analysis		
	Model Identified			Model Identified			Model Identified			Model Identified			Model Identified			Model Identified		
	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2	Mod.	X_1^2	X_2^2
0.02	0.273	0.514	0.165	0.228	0.398	0.153	0.199	0.424	0.103	0.155	0.306	0.092	0.106	0.272	0.038	0.068	0.166	0.031
0.15	0.526	0.864	0.283	0.446	0.760	0.267	0.460	0.823	0.222	0.372	0.698	0.199	0.354	0.729	0.146	0.260	0.574	0.116
0.35	0.607	0.927	0.328	0.524	0.848	0.316	0.548	0.903	0.270	0.456	0.806	0.246	0.449	0.847	0.195	0.346	0.710	0.156

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Note. For models identified, Mod = nonadditive model, X_1^2 = nonlinear X_1 model, X_2^2 = nonlinear X_2 model